ECE 553, Spring 2008 — Problem Set #2

Issued: February 16th, 2008 — **Due: February 25th, 2008**

Primary Reading: Handout 1 & 2 and lecture notes.

Homework is due at the beginning of class on the due day!

Problems:

1. **A simple target-set control problem** (10 points)
   You are given a control system described by

   $$\ddot{x}(t) = u(t), \quad x(0) = -2\sqrt{2}, \dot{x}(0) = 5\sqrt{2},$$

   which is to be steered to the target set $\Gamma = \{(x, \dot{x}) \in \mathbb{R}^2 : x^2 + \dot{x}^2 = 1\}$ (a circle) in unit time while minimizing the control energy

   $$J = \int_0^1 u^2(t)\ dt.$$

   Convert this problem to a calculus of variations problem, and obtain its solution (based on the E-L equation you obtained from the previous homework for problem 2.a).

2. **A free end-time problem** (10 points)
   You are given a control system described by

   $$\ddot{x}(t) = u(t), \quad x(0) = 1, \dot{x}(0) = 1,$$

   which is to be steered to the target set $\Gamma = \{(x, \dot{x}) \in \mathbb{R}^2 : x(t_f) = -t_f^2\}$ (where t_f is free) while minimizing the control energy

   $$J = \int_0^{t_f} u^2(t)\ dt.$$

   Convert this problem to a calculus of variations problem, and obtain its solution (based on the E-L equation you obtained from the previous homework for problem 2.a). What are the optimal values of t_f and $x(t_f)$.

3. **Calculus of variations with final value cost** (10 points)
   Let

   $$J(x) = \psi(x(t_f)) + \int_{t_0}^{t_f} \phi(x(t), \dot{x}(t), t)\ dt$$

   where t_0, t_f are fixed, $x(t_0), x(t_f)$ are free, and ψ, ϕ are at least twice continuously differentiable in their arguments. Consider the problem of minimizing J subject to the mixed end-point constraints

   $$x(t_0) + 2x(t_f) = 1.$$

   If x^o is an optimal solution to this problem, obtain the first-order necessary conditions, along with the boundary (transversality) conditions.

4. (10 points)
Find the extremals of the functional

$$J(x) = \int_0^{\frac{\pi}{2}} [\ddot{x}^2 - x^2 + t^2]\; dt$$

subject to the boundary conditions:

$$x(0) = 1, \quad \dot{x}(0) = 0, \quad x(\pi/2) = 0, \quad \dot{x}(\pi/2) = 1.$$