Homework is due at the beginning of class on the due day!

Problems:

1. **Nonlinear System** (10 points)
   You are given a nonlinear input-output system which satisfies the nonlinear differential equation:
   \[
   \ddot{y}(t) = 2y - (y^2 + 1)(\dot{y} + 1) + u + 2\dot{u}.
   \]
   
   (a) Obtain a nonlinear state-space representation.
   
   (b) Linearize this system around its equilibrium output trajectory when \(u(\cdot) = 0\), and write it in state space form.

2. **Linked Pendulum Problem** (10 points)
   The following systems are useful to model one- or two-line robotic manipulators. For each system:
   
   i. Obtain a nonlinear state-space representation
   
   ii. For \(\theta, \theta_1, \text{ and } \theta_2\) very small, linearize the system, or state why the system can’t be linearized.

   Hint: For the first system, apply Newton’s Law in the tangential direction:
   \[
   u\cos\theta - mg\sin\theta = ml\ddot{\theta}.
   \]
3. **Satellite Problem** (10 points)

Model the earth and satellite as particles. The *normalized* equation of motion can be simplified to 2 dimensions:

\[ \ddot{r} = r\dot{\theta}^2 - \frac{k}{r^2} + u_1 \]  
\[ \ddot{\theta} = -2\frac{\dot{r}\dot{\theta}}{r} + \frac{1}{r}u_2 \]  

with \( u_1, u_2 \) representing the radial and tangential forces due to thrusters. The reference orbit with \( u_1 = u_2 = 0 \) is circular with \( r(t) \equiv p \) and \( \theta(t) = \omega t \). From the first equation it follows that \( p^3\omega^2 = k \) (the Kepler’s law). Obtain the linearized equation about this orbit. (What is the minimum number of state variables?)

4. **A Multiple Input and Multiple Output (MIMO) System** (10 points)

For a system is described by the pair of differential equations:

\[ \dddot{y}_1 + 2\ddot{y}_1 + 3y_2 = u_1 + \dot{u}_1 + \dot{u}_2 \]  
\[ \dddot{y}_2 - 3\ddot{y}_2 + \dot{y}_1 + y_2 + y_1 = u_2 + \dot{u}_2 - u_3 \]  

obtain a state space realization by choosing \( y_1 \) and \( y_2 \) as two of the state variables. (Note: \( y^{(n)} \) denotes the \( n^{th} \) order derivative of \( y \).)

5. **State-space vs. Input-output Representation** (10 points)

A Single Input Single Output (SISO) system is described by the transfer function:

\[ G(s) = \frac{s + 1}{s^3 + 4s^2 + 5s + 2}. \]  

(a) Obtain a three dimensional realization in the controllable canonical state representation.

(b) Obtain a three dimensional realization in the observable canonical state representation.

(c) Use partial fractions to obtain a state representation. Be careful about the repeated root.